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Köppl Turyna, Monika

Agenda Austria

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How asymmetric funding of parties can lead to political polarization

Monika Köppl–Turyna^{1,*}

Agenda Austria, Schottengasse 1/3 1010 Vienna, Austria

Abstract

This work analyzes the impact of asymmetric financial constraints on the platforms of parties using a formal model of elections. Main results show that when a party faces a tight financial constraint the platform chosen in equilibrium is further away from its ideal point compared to the case when the campaign expenses are not limited. Moreover, we have shown that the platform of the party which is facing a tighter financial constraint is further away from its ideal point than of the opponent. These results show the theoretical foundations for the empirical observations made, about the impact of public funding of parties on their platforms.

JEL classification: D72; D78

Keywords: campaign finance, polarization, endogenous valence, public funding

1. Introduction

This work looks at the interrelation between the financial constraints and platforms chosen by the parties competing in a two-party system. When voters vote for any of the two candidates they not only look at the platforms offered, but are also subject to campaign activities, which might increase the chance of voting for any party irrespective of the chosen platforms. Therefore, the parties face a trade-off between choosing a platform appealing to the median voter and spending resources on campaign activities. When financial constraints are introduced on the parties, this trade-off will result in a different choice of platforms compared to the case when the parties are unconstrained.

The most natural institutional setting, in which the parties face financial constraints is

*Corresponding author: Phone: +43 1 361 99 61 20

¹Email: monika.koepl-turyna@agenda.austria.at

associated with the public funding of parties. According to Austin and Tjernström (2003), of 111 analyzed countries, public funding of parties is present in 65 countries. In only 12 of them the public funding is equally distributed, whereas in other cases it is related to the current (19 countries) or past (25 countries) electoral success or current representation in the legislature (25 countries). Therefore, in the majority of the countries which actually finance the parties from the budget, the amount of money at disposal of the parties is one way or the other related to its past electoral performance. In this institutional environment, a natural result is that the parties with past electoral successes will have access to higher amount of financing than others. At the same time, in countries in which public financing is present, parties typically face severe constraints on access to other types of financing, and therefore private funds constitute just a small fraction of the financing obtained from the government.

The phenomenon of the interrelation between public funding presence and choices of platforms by the parties has been observed empirically by Köppl-Turyna (2014). Köppl-Turyna (2014) finds that for the sample of 45 developed countries, parties tend to locate further away from median voter when public financing is present. A natural explanation for this phenomenon is the "barrier to entry" created by the public financing system. Parties with past electoral success have access to substantially higher level of financing and therefore can remain closer to their ideal points, whereas the disadvantaged candidates, are forced to choose divergent platforms. In this work, we attempt at analyzing this relationship with a formal model of elections.

The theoretical literature on public funding of parties is rather scarce. Ortuño Ortin and Schultz (2005) analyze a two-party model in which parties have access to public funding assigned on the base of future electoral success. It is shown that a public funding system increases policy convergence. The effect is larger, the more funding depends on vote shares. If the parties have access to other means of campaign finance given in a lump-sum way,

the effect is moderated. It is important to mention, that the authors analyze a symmetric case.

Troumpounis (2012) analyzes using a two-party group turnout model, and compares the effect of two types of public funding systems on parties mobilization effort and the equilibrium turnout. Allowing one party to have a larger support than the other, the author uncovers differences regarding the equilibrium structure: while in the unique equilibrium of per seat funding systems both parties exert the same amount of effort, a per vote funding system results in an asymmetric equilibrium in which the advantaged party exerts higher effort than its opponent.

Finally, the closest work to this one is Ortín and Schultz (2012) who consider public funding of political parties when some voters are poorly informed about parties' candidates and campaigns are informative. For symmetric equilibria, it is shown that more public funding leads parties to chose more moderate candidates, and that an increase in the dependence of the funding on vote shares induces further moderation and improves welfare. If parties are asymmetric, vote-share-dependent public funding benefits the large party and makes it moderate its candidate, while the smaller party reacts by choosing a more extremist candidate. It is important to notice that the latter results are derived only as numerical simulations.

This work complements the previous approaches in several ways. First of all, the cited works concentrate on the informative aspect of campaigning, whereas we focus on the case when campaigning has a purely persuasive effect. Second of all, we directly consider the case of financial constraints as well as the interrelation of the constraints with other variables in the model, such as the policy importance. Finally, although with a simpler model than the mentioned literature, we derive analytical results in the case of asymmetric equilibria.

This work is structured as follows. The next section presents the structure of the model.

Section 3 describes the analytical results. Section 4 concludes. More complex derivations are presented in the Appendix.

2. The model

The structure of the presented model is to some extent derived from the work of Herrera et al. (2008), yet with important differences that allow to analyze the research question. Two policy-motivated parties L and R compete in a first-past-the-post setup. Decisions of the parties involve two components: the choice of a binding policy platform, and a choice of campaigning efforts. The platforms l and r are simultaneously chosen in the first stage and after observing the policy choices, the campaign efforts L and R are simultaneously chosen in the second stage. After the platforms and campaign efforts have been announced, voting takes place. Parties' utility functions contain utility from obtaining the office, denoted B , which includes gain from winning the election such as perks from office, as well as disutility from a policy that will be implemented after the election, if it does not exactly correspond to the parties' ideal points. We assume that B is a lump-sum benefit, as this assumption matches well with the first-past-the-post set-up, in which the size of the perks from office will be less dependent on the actual margin of victory, than in a proportional representation system. Specifically, the payoffs functions are:

$$U^L = \begin{cases} B - l - L & \text{if L wins} \\ -(1 - r) - L & \text{if R wins} \end{cases} \quad (1)$$

and

$$U^R = \begin{cases} B - r - R & \text{if R wins} \\ -(1 - l) - R & \text{if L wins.} \end{cases} \quad (2)$$

The net payoffs include the benefits from office B , the ideological costs r and l as well as the linear cost of campaign activities L and R .

The outcome of the election is determined by the voters, who are uniformly distributed on a unit interval $v \in [0, 1]$. Voters' preferences are a function of the policy distance between their ideal points, an idiosyncratic party bias b as well as a bias towards party L b_v . The voter with an ideal point v prefers party L whenever

$$-a|v - l| + b + b_v > -a|v - (1 - r)|, \quad (3)$$

and party R if the inequality is reversed. The idiosyncratic bias b is uniformly distributed on $[-\beta, \beta]$, where β will be assumed sufficiently high as not to predict with probability one the winner of the election. The parameter $a \geq 1$ in the utility of the voter measures the importance of the policy message relative to the idiosyncratic as well as party biases, that is it reflects the importance of the policy dimension to the voter.

The party bias b_v is uniformly distributed on $[-\alpha, \alpha]$, where α is assumed low enough, so that it is impossible to predict the results of the election in advance, that is $\alpha + 1 < \beta$. We model the campaign activity technology similarly to Herrera et al. (2008), yet under the assumption of perfect targeting, in order to concentrate on the main effects. This assumption might seem strict, but allowing for imperfect targeting does not change any of the main conclusions and makes analytical representation of the results much more complex. Campaign activity linearly increases the probability of winning for each party, i.e. for party L , the final probability of winning would equal $L \times P(L)$, where $P(L)$ is the fraction of voters voting for L derived from (3) as explained below.

The most important assumption in our model is the fact that parties can be financially constrained. That is, we introduce parameters Θ_L and Θ_R , which are the financial

constraints of parties L and R respectively. That is, in any equilibrium it holds

$$L \leq \Theta_L$$

$$R \leq \Theta_R$$

and we assume without loss of generality that $\Theta_L > \Theta_R$.

The timing of the game is as follows:

1. The parties simultaneously choose the positions l and r .
2. The parties simultaneously choose the level of spending L and R given the budget constraint.
3. Nature draws b_v and b .
4. Voting takes place.

From (3) it follows, that the probability that voter v favors party L equals

$$P(v \text{ favors } L) = \begin{cases} P_1(L) = \frac{a+b-al-ar+\beta}{2\beta} & \text{if } v \in [0, l] \\ P_2(L) = -\frac{av}{\beta} + \frac{a+b+al-ar+\beta}{2\beta} & \text{if } v \in (l, 1-r] \\ P_3(L) = \frac{-a+b+al+ar+\beta}{2\beta} & \text{if } v \in (1-r, 1]. \end{cases} \quad (4)$$

Overall fraction of voters in favor of L equals

$$P(L) = \int_0^l P_1 dv + \int_l^{1-r} P_2 dv + \int_{1-r}^1 P_3 dv = \frac{(b-a(l-r))(-1+l+r)+\beta}{2\beta}. \quad (5)$$

Given the linear campaigning technology, the overall probability that L wins the election is given by

$$L \times P(L) > R \times (1 - P(L)) \quad (6)$$

where $L, R \in [0, 1]$, or equivalently that

$$b > a(l-r)(-1+l+r) + \frac{(-L+R)\beta}{L+R}. \quad (7)$$

Denoting F_b the distribution of b , the expression above equals

$$1 - F_b(\hat{b}) = 1 - \frac{a(l-r)(-1+l+r) + \alpha + \frac{(-L+R)\beta}{L+R}}{2\alpha}. \quad (8)$$

In the second stage, the parties simultaneously choose their campaign activities level. Given (1), (2) and (8), the expected payoffs of parties L and R are

$$\pi_L = [1 - F_b(\hat{b})](B - l) + F_b(\hat{b})(-(1 - r)) - L \quad (9a)$$

$$\pi_R = F_b(\hat{b})(B - r) + [1 - F_b(\hat{b})](-(1 - l)) - R. \quad (9b)$$

The corresponding Lagrange functions are

$$L_L = \pi_L - \lambda_1(L - \Theta_L) \quad (10a)$$

$$L_R = \pi_R - \lambda_2(R - \Theta_R), \quad (10b)$$

and the corresponding Kuhn–Tucker conditions are

$$\frac{\partial L_L}{\partial L} = 0 \quad \frac{\partial L_R}{\partial R} = 0 \quad (11a)$$

$$\frac{\partial L_L}{\partial \lambda_1} \leq 0 \quad \frac{\partial L_R}{\partial \lambda_2} \leq 0 \quad (11b)$$

$$\lambda_1(L - \Theta_L) = 0 \quad \lambda_2(R - \Theta_R) = 0 \quad (11c)$$

$$\lambda_1 \geq 0 \quad \lambda_2 \geq 0. \quad (11d)$$

3. Results

We shall analyze the possible cases in turn.

3.1. Case 1: Unconstrained solution: $L^* < \Theta_L$ and $R^* < \Theta_R$

We will consider the unconstrained solution to serve as a benchmark for the subsequent results. In this case, the optimal expenditure level is below the financial constraint for both parties, that is neither constraint is binding. In this case we have

$$L^* = R^* = \frac{\beta(1 - l - r + B)}{4\alpha}$$

and $\lambda_1 = \lambda_2 = 0$. If $1 - l - r + B \leq 0$ the expenditure of both parties in equilibrium equals zero, the case which will not further analyze; we assume, therefore here and throughout the next sections that $1 - l - r + B > 0$ to avoid dealing with the uninteresting corner solutions. Otherwise, the level of expenditure is given by L^* and R^* , the first stage first order conditions are symmetric and the platforms chosen in equilibrium are

$$p_{uncons}^* = l_{uncons}^* = r_{uncons}^* = \frac{1}{4} \left(2 + B - \frac{\sqrt{a(aB^2 + 4\alpha - 2\beta)}}{a} \right) \quad (12)$$

It is easy to see that in this case we have

$$\frac{\partial p_{uncons}^*}{\partial B} > 0$$

and

$$\frac{\partial p_{uncons}^*}{\partial a} \begin{matrix} \leq \\ > \end{matrix} 0,$$

dependent on the relation between α and β , i.e. if $2\alpha - \beta > 0$ then $\partial p_{uncons}^* / \partial a > 0$. We conclude therefore that the relation between the platforms of parties in equilibrium and the importance of policy compared to the stochastic components depends on the strength

of the latter. If $\alpha + 1 < \beta < 2\alpha$, that is the idiosyncratic bias is not too large, parties converge to the median along with increasing a , and the opposite holds if the condition is not satisfied. This result is intuitive: when the policy dimension is important to the voters, parties gain support through the movement in the direction of the median voter. If the voters are easily impressionable, that is a is low, parties prefer to invest in the campaigning efforts and simultaneously bear a lower policy cost. *Polarization* defined as $1 - r - l$ is *decreasing* in the benefit from holding office and the effect of the policy importance is ambiguous, as explained above.

3.2. Case 2: Constrained solution: $L^* = \Theta_L$ and $R^* = \Theta_R$

In this case we have

$$\lambda_1 = -1 + \frac{(1 - l + B - r)\beta\Theta_R}{\alpha(\Theta_L + \Theta_R)^2} \quad (13a)$$

$$\lambda_2 = -1 + \frac{(1 - l + B - r)\beta\Theta_L}{\alpha(\Theta_L + \Theta_R)^2}, \quad (13b)$$

and the dual feasibility conditions place additional constraints on the parameter values. $\lambda_2 > \lambda_1$ because $\Theta_L > \Theta_R$, and the necessary condition for the solution to be dually feasible is

$$\Theta_R \leq \frac{\beta(1 - l - r + B)}{4\alpha}.$$

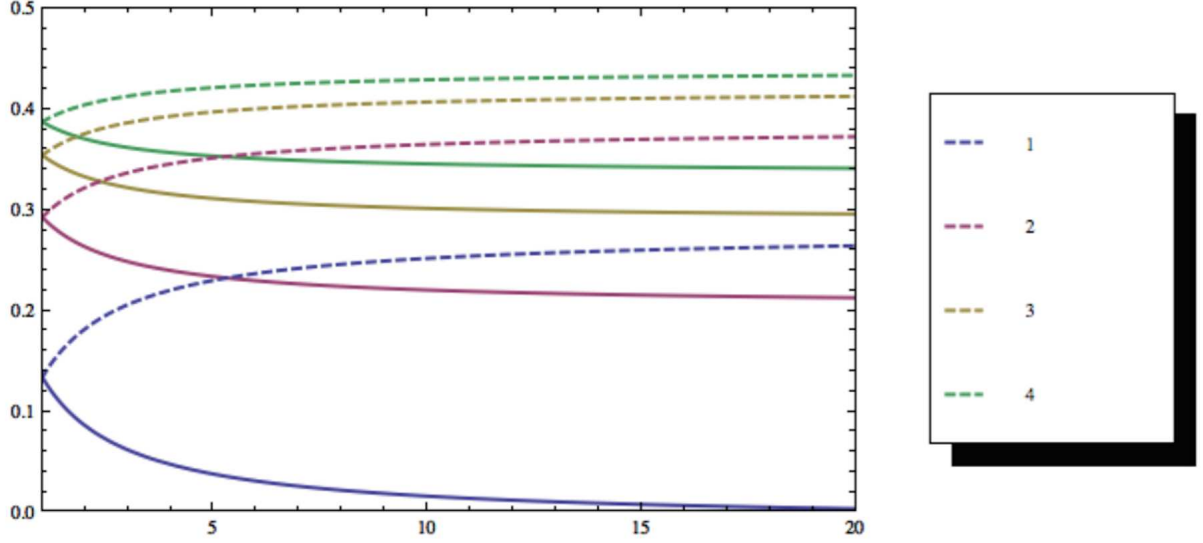
The first-stage solutions to the constrained problem are

$$l_{cons}^* = \frac{1}{2} + \frac{B}{4} - \frac{\sqrt{a(aB^2 + 4\alpha)}(2\beta(\Theta_L - \Theta_R) + (aB^2 + 4\alpha)(\Theta_L + \Theta_R))}{4a(aB^2 + 4\alpha)(\Theta_L + \Theta_R)} \quad (14a)$$

$$r_{cons}^* = l_{cons}^* + \frac{\beta(\Theta_L - \Theta_R)}{\sqrt{a(aB^2 + 4\alpha)}(\Theta_L + \Theta_R)}. \quad (14b)$$

Proposition 1. *The platform of party L is strictly closer to its ideal point than the platform of party R, iff $\Theta_L - \Theta_R > 0$.*

Figure 1: Platforms of parties for different values of a and increasing difference in budgets.



Proof. Brief inspection of (14a) reveals that $r_{cons}^* > l_{cons}^*$ iff $\Theta_L - \Theta_R > 0$. \square

The financially advantaged party always chooses a point closer to its ideal point than the financially disadvantaged opponent. Figure 3.2 presents simulations of the analytical results for varying values of parameters a and Θ_R .

Figure 3.2 visualizes the choices of platforms given diverse policy importance parameters (a equals 1,2,3 or 4) as well as increasing difference in budgets: Θ_R is kept fixed at 1 whereas Θ_L varies from 1 to 20. Position of party L is always depicted in solid line, position of party R in dashed line. We can observe that when a increases, both platforms converge to the median and the difference between them decreases. On the other opposite, for a low value of a and a high difference in platforms, party L barely moves away from its ideal point, and party R strongly converges.

Replacing for l and r in the expressions of the Lagrange multipliers yields

$$\lambda_1 = \frac{\sqrt{a(aB^2 + 4\alpha)}\beta\Theta_R + a(B\beta\Theta_R - 2\alpha(\Theta_L + \Theta_R)^2)}{2a\alpha(\Theta_L + \Theta_R)^2} \quad (15a)$$

$$\lambda_2 = \frac{\sqrt{a(aB^2 + 4\alpha)}\beta\Theta_L + a(B\beta\Theta_L - 2\alpha(\Theta_L + \Theta_R)^2)}{2a\alpha(\Theta_L + \Theta_R)^2}, \quad (15b)$$

A sufficient condition for a solution in which both parties are constrained to be feasible is

$$B \geq \frac{-\beta^2\Theta_L^2 + a\alpha(\Theta_L + \Theta_R)^4}{a\beta\Theta_L(\Theta_L + \Theta_R)^2},$$

thus either the benefits from holding office are high enough or the policy importance is low enough. In this case we have

$$\frac{\partial l_{cons}^*}{\partial a} = \frac{aB^2(\beta(\Theta_L - \Theta_R) + \alpha(\Theta_L + \Theta_R)) + 2\alpha(\beta(\Theta_L - \Theta_R) + 2\alpha(\Theta_L + \Theta_R))}{2(a(aB^2 + 4\alpha))^{3/2}(\Theta_L + \Theta_R)} > 0$$

$$\frac{\partial r_{cons}^*}{\partial a} = \frac{aB^2(\beta(-\Theta_L + \Theta_R) + \alpha(\Theta_L + \Theta_R)) + 2\alpha(\beta(-\Theta_L + \Theta_R) + 2\alpha(\Theta_L + \Theta_R))}{2(a(aB^2 + 4\alpha))^{3/2}(\Theta_L + \Theta_R)}.$$

The sign of the latter derivative depends on the parameters of the model; it can be shown that as long as

$$\alpha > \frac{\beta(\Theta_L - \Theta_R)}{\Theta_L + \Theta_R},$$

r_{cons}^* is increasing in a . Therefore, similarly to the unconstrained case, the effect of the policy importance depends on the magnitude of the stochastic components of voters' utility functions. Unlike in the symmetric case, however, the advantaged party always converges to the median voter if the policy importance is high. The disadvantaged opponent will converge to the median along with a only if α is high comparatively to the difference in budgets. If $\Theta_L - \Theta_R$ is high, this condition will be correspondingly more difficult to be satisfied: more a high difference in budgets, the disadvantaged party is likely to diverge

from the median voter even if the policy importance is high.

Polarization is strictly decreasing in a as

$$\frac{\partial(1 - l_{cons}^* - r_{cons}^*)}{\partial a} = -\frac{\alpha}{a\sqrt{a(aB^2 + 4\alpha)}} < 0.$$

Polarization is also decreasing in B given the assumptions, as

$$\frac{\partial(1 - l_{cons}^* - r_{cons}^*)}{\partial B} = \frac{1}{2} \left(-1 + \frac{aB}{\sqrt{a(aB^2 + 4\alpha)}} \right) < 0.$$

Finally, the platforms change along with the difference in budget constraints $Dif\Theta \equiv \Theta_L - \Theta_R$ according to:

$$\frac{\partial l_{cons}^*}{\partial Dif\Theta} = -\frac{\sqrt{a(aB^2 + 4\alpha)}\beta}{2a(aB^2 + 4\alpha)(\Theta_L + \Theta_R)} < 0$$

$$\frac{\partial r_{cons}^*}{\partial Dif\Theta} = \frac{\sqrt{a(aB^2 + 4\alpha)}\beta}{2a(aB^2 + 4\alpha)(\Theta_L + \Theta_R)} > 0$$

and

$$\frac{\partial(1 - l_{cons}^* - r_{cons}^*)}{\partial Dif\Theta} = 0.$$

From the symmetry on the model follows that polarization is not changing along with the difference in the budget available, as the changes offset each other. Yet, *the difference* in the platform chosen defined $r^* - l^*$ does and it holds

$$\frac{\partial r_{cons}^* - l_{cons}^*}{\partial Dif\Theta} = \frac{2\beta\Theta_R}{\sqrt{a(aB^2 + 4\alpha)}(\text{Dif}\Theta + 2\Theta_R)^2} > 0,$$

that is the disadvantaged party R chooses a platform strictly further from its ideal point than party L , and the difference is increasing in the difference in the budgets of the two parties.

Proposition 2. *The platform of l^* in the constrained case is strictly closer to the party's ideal point than in the unconstrained case, i.e., $l_{uncons}^* - l_{cons}^* > 0$, whereas for party R , $r_{uncons}^* - r_{cons}^* < 0$ iff $3\Theta_R < \Theta_L$.*

Proof. Proof can be found in the Appendix. \square

Proposition 2 states that the platform of the party with access to more financing is closer to its ideal point than in the unconstrained case, whereas for the disadvantaged party the result is ambiguous. The constrained platform is further away from the ideal point of R when the difference in the budgets of the two parties is high. Consequently, existence of a high difference in the access to financing between parties L and R generates the result which has been observed empirically: the advantaged party can compete in the election remaining close to its ideal points, whereas the disadvantaged party is forced to adopt a divergent position. Therefore, not only does the disadvantaged party almost certainly lose the election, but additionally it bears a high policy cost. We visualize this result in Figure 3.2.

Figure 3.2 shows, that if the difference in budgets is high, the advantaged party L locates much closer to its ideal point compared to the unconstrained case, whereas party R is further disadvantaged in term of policy costs: it locates further away from its ideal point than in the unconstrained case.

Finally, analysis of the second derivatives reveals how financial advantage of one party interacts with the parameter a describing the importance of policy for the voters. Namely, it holds,

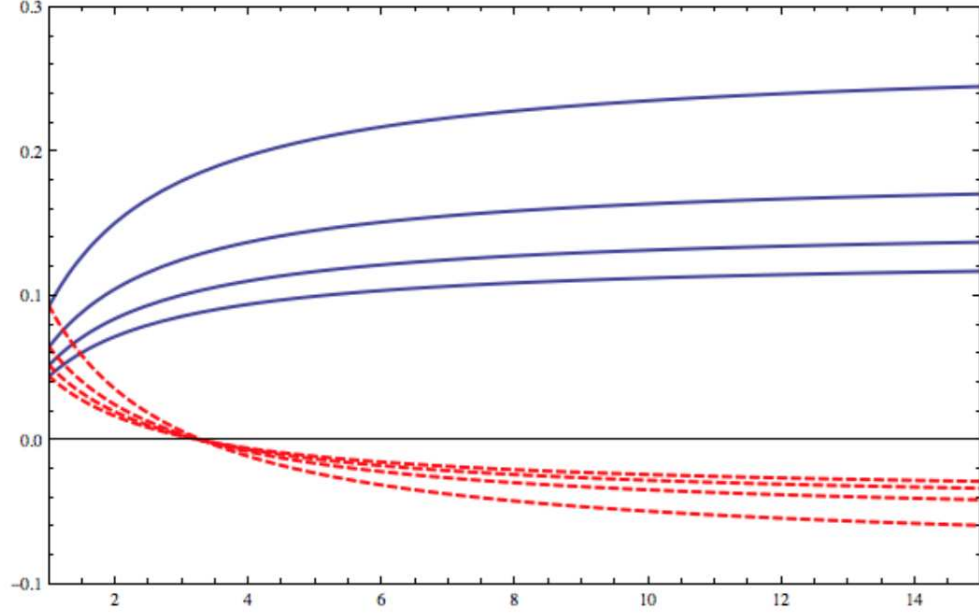
$$\frac{\partial^2 l_{cons}^*}{\partial a \partial \Theta_L} > 0$$

and

$$\frac{\partial^2 r_{cons}^*}{\partial a \partial \Theta_L} < 0$$

that is, holding Θ_R constant, movement of the platform l towards the median caused by

Figure 2: Difference between l_{uncons}^* and l_{cons}^* (blue) and r_{uncons}^* and r_{cons}^* (red) and increasing difference in budgets.



the financial advantage is *stronger* for high values of a and the effect is opposite for the platform r .

3.3. Case 3: L^* unconstrained and $R^* = \Theta_R$

An interesting case arises when one the disadvantaged party is financially constrained, whereas the advantaged party can use the best response to the constrained opponent's expenditure. In this case we have

$$R_{Rcons}^* = \Theta_R \tag{16a}$$

$$L_{Rcons}^* = -\Theta_R + \frac{\sqrt{(1-l+B-r)\alpha\beta\Theta_R}}{\alpha} \tag{16b}$$

A necessary condition that guarantees positive level of expenditure as well as dual feasibility is

$$\Theta_R \leq \bar{\Theta}_R \equiv \frac{\beta(1-l+B-r)}{\alpha}.$$

The first-stage first order conditions are

$$0 = \frac{a(1+3l^2+B-2l(2+B-r)-r^2)-\alpha+\beta\left(-1+\frac{2\alpha\Theta_R}{\sqrt{(1-l+B-r)\alpha\beta\Theta_R}}\right)}{2\alpha} \quad (17a)$$

$$0 = \frac{a(1-l^2+B+2lr-2(2+B)r+3r^2)-\alpha+\beta-\frac{\alpha\beta\Theta_R}{\sqrt{(1-l+B-r)\alpha\beta\Theta_R}}}{2\alpha} \quad (17b)$$

for L and R respectively. Solving explicitly the above system is possible but yields complex formulas which do not provide deep insight as for direction of the effects. However, the characteristics of the equilibrium can be analyzed. Firstly we prove that the equilibrium exists. Thus, we show that the profits are concave in the platform choice for the admissible parameter values.

Corollary 1. *The profit functions are concave in the own choice of platforms l and r if $l_{Rcons}^*, r_{Rcons}^* \leq 1/2$ and $a \geq \frac{\beta}{4B^2}$.*

Proof can be found in the Appendix.

Secondly, we need to show that indeed the platforms in equilibrium satisfy $l^*, r^* \leq 1/2$.

Corollary 2. *In equilibrium $l_{Rcons}^*, r_{Rcons}^* \leq 1/2$ if $\Theta_R \geq \frac{\beta-\alpha}{\beta+\alpha}\Theta_L$.*

Proof can be found in the Appendix.

If the difference between the budgets of the two parties is not too big, both platforms are lower than $1/2$. In fact, the platform of the advantaged party L is always lower than $1/2$ as proven in the Appendix. On the other hand, for a very significant difference between Θ_L and Θ_R the platform of party R would be on the same side of the median voter position as party's L . Now we can turn to the analysis of the properties of the equilibrium. First

of all notice that since the program of party L is more relaxed, the platform chosen in equilibrium will be closer to its ideal point, that is $l_{Rcons}^* < l_{cons}^*$.

Using the implicit function theorem we can analyze the impact of Θ_R on the optimal platform of party R (all expressions in the appendix):

$$\frac{\partial r_{Rcons}^*}{\partial \Theta_R} > 0, \quad (18)$$

if $a < \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}$ and the value of a here determined is always positive. Similarly for party L we have

$$\frac{\partial l_{Rcons}^*}{\partial \Theta_R} > 0, \quad (19)$$

if $a > \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}$. For any level of a , the change in the budget of party R will have opposite effects on the platforms of L and R .

The effect of policy importance is for party L

$$\frac{\partial l_{Rcons}^*}{\partial a} > 0 \quad (20)$$

iff

$$a > \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}$$

and for party R

$$\frac{\partial r_{Rcons}^*}{\partial a} > 0 \quad (21)$$

iff

$$a < \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}.$$

Therefore, similarly to the expressions for the impact of Θ_R , for all values of a the derivatives have opposite signs: for high values of the policy importance, the platform of L

converges to the median and platform of party R diverges; the opposite holds for low values of a .

4. Conclusions

In this work we have presented a model of elections in which parties are financially constrained. Main results show that when a party faces a tight financial constraint the platform chosen in equilibrium is further away from its ideal point compared to the case when the campaign expenses are not limited. Moreover, we have shown that the platform of the party which is facing a tighter financial constraint is further away from its ideal point than of the opponent. These results show the theoretical foundations for the empirical observations made, about the impact of public funding of parties on their platforms. Some of the results exhibit nonlinearities, which have not yet been observed in other theoretical works on public financing. These results should be further verified by means of empirical studies.

Appendix

Proof of Proposition 2. The difference $l_{uncons}^* - l_{cons}^*$ equals

$$\frac{1}{4} \left(-\frac{\sqrt{a(aB^2 + 4\alpha - 2\beta)}}{a} + \frac{2\beta(\Theta_L - \Theta_R) + (aB^2 + 4\alpha)(\Theta_L + \Theta_R)}{\sqrt{a(aB^2 + 4\alpha)}(\Theta_L + \Theta_R)} \right)$$

and is equal to zero iff $\beta = 0$ or $\beta = -\frac{(aB^2 + 4\alpha)(3\Theta_L - \Theta_R)(\Theta_L + \Theta_R)}{2(\Theta_L - \Theta_R)^2} < 0$. Since $l_{uncons}^* - l_{cons}^*$ is increasing in β as long $\Theta_L > \Theta_R$ it follows that the difference is positive.

It holds

$$r_{cons}^* - r_{uncons}^* > 0 \quad \text{if} \quad 3\Theta_R < \Theta_L \quad \text{and} \quad \beta \leq \frac{(aB^2 + 4\alpha)(\Theta_L - 3\Theta_R)(\Theta_L + \Theta_R)}{2(\Theta_L - \Theta_R)^2}.$$

The second condition is equivalent to $\beta > 0$ if the first condition is satisfied, therefore $3\Theta_R < \Theta_L$ is sufficient for the constrained solution to be further away from the ideal point of R than the unconstrained solution. \square

Proof of Corollary 1. The second order conditions are:

$$\text{SOC}_l = \frac{2a(-2 + 3l - B + r) + \frac{\alpha^2 \beta^2 \Theta_R^2}{((1-l+B-r)\alpha\beta\Theta_R)^{3/2}}}{2\alpha} \quad (22a)$$

$$\text{SOC}_r = \frac{2a(-2 + l - B + 3r) - \frac{\alpha^2 \beta^2 \Theta_R^2}{2((1-l+B-r)\alpha\beta\Theta_R)^{3/2}}}{2\alpha} \quad (22b)$$

The second order condition for party R is always satisfied if $l, r \leq 1/2$ and $\Theta_R \leq \bar{\Theta}_R$. For party L we have

$$\text{SOC}_l(\bar{\Theta}_2) = \frac{4a(-2 + 3l - B + r) + \frac{\beta}{1-l+B-r}}{4\alpha}. \quad (23)$$

and since

$$\frac{\partial \text{SOC}_l}{\partial \Theta_R} = \frac{\alpha \beta^2 \Theta_R}{4((1-l+B-r)\alpha\beta\Theta_R)^{3/2}} > 0, \quad (24)$$

the second order condition is strictly increasing in Θ_R . For $\bar{\Theta}_R$ the second order condition is satisfied if

$$4a \geq \frac{\beta}{B^2} \quad (25)$$

for $l = r = 1/2$ and since the second order conditions are strictly increasing and jointly increasing in l and r , it follows that the problem is concave. \square

Proof of Corollary 2. It is easy to see that platform l in this case satisfies $l \leq 1/2$. Since the problem in this case is more relaxed for party L than in case when both parties are financially constrained, the platform chosen by L will be weakly closer to its ideal point, than in Case 2. One can also notice that the first order condition for Case 3 evaluated at the optimal solution to Case 2 is negative. In the case when both parties are financially constrained $l_{cons}^* \leq 1/2$ iff $\Theta_L > \Theta_R$ which completes the proof.

For the platform r , we can look at the first order condition. Evaluated for any platform l at $r = 1/2$ the first order condition is negative as long as $\Theta_R \geq \frac{\beta-\alpha}{\beta+\alpha}\Theta_L$. Since the problem is concave for $r \leq 1/2$, it follows that the optimal r^* in this case has to be lower than $1/2$. \square

Comparative statics of l_{Rcons}^, r_{Rcons}^* wrt a and Θ_R .*

$$\begin{aligned} \frac{\partial r_{Rcons}^*}{\partial \Theta_R} &= ((-1 + l - B + r)^2(-2 + l - B + 3r)\alpha\beta) \\ &\quad / \left((-2 + 2l - B + 2r)\sqrt{-(-1 + l - B + r)\alpha\beta\Theta_R} \right. \\ &\quad \left. \left(4a(-1 + l - B + r)^2(-2 + 2l - B + 2r) + \sqrt{-(-1 + l - B + r)\alpha\beta\Theta_R} \right) \right) > 0, \quad (26) \end{aligned}$$

if $a < \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}$ and the value of a here determined is always positive. Similarly for party L we have

$$\begin{aligned} \frac{\partial l_{Rcons}^*}{\partial \Theta_R} &= (-1 + l - B + r)(-4 + 3l - 2B + 5r)\sqrt{-(-1 + l - B + r)\alpha\beta\Theta_R} \\ &\quad / (-2 + 2l - B + 2r)\Theta_R \left(4a(-1 + l - B + r)^2(-2 + 2l - B + 2r) + \sqrt{-(-1 + l - B + r)\alpha\beta\Theta_R} \right) > 0, \quad (27) \end{aligned}$$

if $a > \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}$. For any level of a , the change in the budget of party R will have opposite effects on the platforms of L and R .

The effect of policy importance is for party L

$$\begin{aligned} \frac{\partial l_{Rcons}^*}{\partial a} = & \left(-4a(-1+l-B+r)^2 \left(-2+4l^3-B(3+B)+2l(2+B-2r)(2+B-r)+ \right. \right. \\ & \left. \left. r+4Br-(2+B)r^2+l^2(-5(2+B)+8r) \right) + \right. \\ & \left. (l^2+3(1+B)-2l(2+B-3r)-4(2+B)r+5r^2) \sqrt{-(-1+l-B+r)\alpha\beta\Theta_R} \right) / \\ & \left(2a(-2+2l-B+2r) \left(4a(-1+l-B+r)^2(-2+2l-B+2r) + \sqrt{-(-1+l-B+r)\alpha\beta\Theta_R} \right) \right) \end{aligned} \quad (28)$$

and for party R

$$\begin{aligned} \frac{\partial r_{Rcons}^*}{\partial a} = & \left(4a(-1+l-B+r)^2 \left(2+3B+B^2-4l(1+B)+l^2(2+B-4r)+ \right. \right. \\ & \left. \left. l(2+B)r-2(2+B)^2r-8lr^2+5(2+B)r^2-4r^3 \right) - \right. \\ & \left. (l^2+3(1+B)-2l(2+B-3r)-4(2+B)r+5r^2) \sqrt{-(-1+l-B+r)\alpha\beta\Theta_R} \right) / \\ & \left(2a(-2+2l-B+2r) \left(4a(-1+l-B+r)^2(-2+2l-B+2r) + \sqrt{-(-1+l-B+r)\alpha\beta\Theta_R} \right) \right). \end{aligned} \quad (29)$$

The first term is positive iff

$$a > \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}$$

and the second is positive iff

$$a < \frac{\sqrt{-\frac{\alpha\beta\Theta_R}{(-1+l-B+r)^3}}}{8-8l+4B-8r}.$$

□

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